1.1 The Characteristics of a Function

Goal: Identify the difference between a relation and a function.

A **RELATION** is a set of ordered pairs. These can be represented in various ways.

**Examples of Relations:**

A) \{ (1,2), (5,3), (9,4), (7,1) \}  
   ...as a **mapping diagram**  
   (see pg 6 for definition)

B) \{ (1,3), (4,2), (3,2), (6,5) \}  
   ...as a **table of values**

C) \{ (1,4), (3,2), (5,4), (3,1) \}  
   ...as a **scatter plot**

The **DOMAIN** is the set of first elements of the ordered pairs (the set of distinct x values)

The **RANGE** is the set of second elements of the ordered pairs (the set of distinct y values)

For each example above we can write the Domain and Range using **SET NOTATION**.

Example A) has… Domain = \{ \text{x} \in \mathbb{Z} | x = 1, 5, 7, 9 \}  and  Range = \{ \text{y} \in \mathbb{Z} | y = 1, 2, 3, 4 \}

Example B) has… Domain = \{ \text{x} \in \mathbb{Z} | x = 1, 3, 4, 6 \}  and  Range = \{ \text{y} \in \mathbb{Z} | y = 2, 3, 45 \}

Example C) has… Domain = \{ \text{x} \in \mathbb{Z} | x = 1, 3, 5 \}  and  Range = \{ \text{y} \in \mathbb{Z} | y = 1, 2, 4 \}

A **FUNCTION** is a relation where each value in the **domain** corresponds to exactly **ONE** element of the **range**. It can also be thought of as a **rule** that associates each x value with only **ONE** y-value.

**Note**: More than one x-value can correspond to the same y-value.  (See pg 7 for an alternate definition)

A relation is **NOT** a function if **one x value has 2 different y-values associated** with it.

- In the examples above, examples A) and B) are **functions**.
- Example C) is **not** a function since the x-value 3 is associated with **two** y-values… y = 1 and y = 2

To visualize this, complete a MAPPING diagram for example C):

Write each **UNIQUE x-value in order from smallest to largest**
All functions are relations, but not all relations are functions. An easy way to determine whether or not a relation is a function is to use its graph and a vertical line test.

The **Vertical Line Test** states that a relation is a function if, for any value of x, you can draw a vertical line through at most **ONLY ONE** point on the graph of the relation.

Consider these examples…

<table>
<thead>
<tr>
<th>Parabola Opening Up</th>
<th>Parabola Opening to the Left</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Parabola Opening Up" /></td>
<td><img src="image2" alt="Parabola Opening to the Left" /></td>
</tr>
</tbody>
</table>
| \( D = \{ x \in \mathbb{R} \} \)  
| \( R = \{ y \in \mathbb{R} \mid y \geq -3 \} \)  |
| \( D = \{ x \in \mathbb{R} \mid x \leq 5 \} \)  
| \( R = \{ y \in \mathbb{R} \} \)  |
| Does this pass the Vertical Line Test? **YES** / **NO**  
| Therefore, is this relation a function? **YES** / **NO**  |
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| Therefore, is this relation a function? **YES** / **NO**  |

<table>
<thead>
<tr>
<th>Straight Line</th>
<th>Sine Function (you’ll see this later)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Straight Line" /></td>
<td><img src="image4" alt="Sine Function" /></td>
</tr>
</tbody>
</table>
| \( D = \{ x \in \mathbb{R} \} \)  
| \( R = \{ y \in \mathbb{R} \} \)  |
| \( D = \{ x \in \mathbb{R} \} \)  
| \( R = \{ y \in \mathbb{R} \mid -1 \leq y \leq 1 \} \)  |
| Does this pass the Vertical Line Test? **YES** / **NO**  
| Therefore, is this relation a function? **YES** / **NO**  |
| Does this pass the Vertical Line Test? **YES** / **NO**  
| Therefore, is this relation a function? **YES** / **NO**  |

Let’s examine the examples from the textbook as well. Open your books to page 8 …

**Classwork / Homework:**
- pg 13 #1ac, 2bcd, 3bc
- pg 14 #5 – 7, 9, 11ab, 16